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**ICS 2018 Problem Sheet #6**

Problem 6.1: completeness of → and ¬

Proof that the two elementary Boolean functions → (implication) and ¬ (negation) are universal, i.e., they are sufficient to express all possible Boolean functions.

Here

For A ,B we have Negation

|  |  |  |
| --- | --- | --- |
| *A* | *B* |  |
| 0 | 0 | 1 |
| 0 | 1 | 1 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

|  |  |
| --- | --- |
|  |  |
| 0 | 1 |
| 1 | 0 |

Now

So,

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *A* | *B* |  |  | *)* | *)* | *)* |
| 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 | 0 | 1 |

Therefore, from the table we can observe that:

*)* is same as

*)* is same as

Since the functions AND and OR can be expresses by the elementary Boolean functions and negation (. We can find the other Boolean functions with these functions.

For Equivalence:

*)* → *))*

For Exclusive OR

*)* → *)))*

For NOT AND

)

For NOT OR

)

Problem 6.2: conjunctive and disjunctive normal form

Consider the following Boolean formula:

1. How many interpretations of the variables P, Q, R and S satisfy ϕ? Provide a proof for your answer.

Here

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | Q | R | S |  |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

Total no interpretations is 24 = 16.

So has Boolean value 1 only in 2 interpretations, on the rest 14 interpretation is 0.

So

1. Given the interpretations that satisfy ϕ, write the formula for ϕ in disjunctive normal form (DNF).

So here we have two conditions where

That are when

So

When we have,

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | Q | R | S |  |
| 1 | 1 | 1 | 1 | 1 |

(

When we have,

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| P | Q | R | S |  |
| 0 | 0 | 0 | 0 | 1 |

(

So by DNF

We have

1. Using the equivalence laws for Boolean expressions, derive the DNF representation of ϕ algebraically from the CNF representation. Write the derivation down stepwise.

Here we have CNF:

Using Distributive Property, we derive,

Converting from CNF to DNF in steps

We know for for any

Therefore, we will be using this throughout to get the solution;

We have

For

For

For

=

=

=

=

Using Associative Property, we have simplified

=

Here all the terms except and have the value 0 because of this property [ .

Therefore the DNF form is: